THE TR-20 ANALOG COMPUTER

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CE 550A

DESCRIPTION OF APPARATUS

EAI TR-20 ANALOG COMPUTER

A. Introduction

The EAI TR-20 is a general purpose desktop analog computer. It contains a maximum of 20 amplifiers and 24 potentiometers. The TR-20 may be connected (slaved) with other TR-20's to increase its capacity and capability.

It can perform the following linear and non-linear mathematical operations:

1. Multiplication by a constant
2. Inversion (sign change)
3. Algebraic summation
4. Continuous integration
5. Multiplication and division of variables
6. The generation of arbitrary functions

B. Linear Components

1. Potentiometers

Potentiometers (pots) are adjustable resistors. There are two types found on the TR-20 i.e. grounded and ungrounded.

Grounded potentiometers are used with a reference voltage to obtain a fixed voltage less than the original reference voltage, or to multiply a problem variable by a constant less than unity.

Ungrounded pots are generally used with nonlinear components, however, there are some instances when they are of use in linear operation. They may be grounded and used as a grounded pots, or they may be used to add (1-K)E to an input or output where K is a potentiometer setting and E is a reference voltage. A schematic of grounded and ungrounded pots are shown in figure 1. (Page 6)

2. Operational Amplifiers

The amplifier is the basic unit of analog computers. Amplifiers may be used to perform inversion, summation, multiplication and integration. When connected with special networks it can be used to perform various non-linear functions such as multiplication, function generation, etc.

The input-output relationship of an operational amplifier is solely dependent on the ratios of the feedback impedance to the input impedance.

$$e_o = - \frac{Z_f}{Z_i} e_i$$

where e_o is the output voltage

Z_f is the feed back impedance

Z_i is the input impedance

e_i is the input

(Figure 2-a)

a. Amplifier as an inverter

The amplifier is used as an inverter when both the input and feedback impedances are resistors of the same size.

$$e_o = - \frac{R_f}{R_i} e_i = - (1) e_i \qquad \text{(Figure 2-b)}$$

b. Amplifier as multiplier of input by a constant.

The amplifier is used as a multiplier when the input and the feedback resistors are not equal. If $R_f > R_i$ then $e_o > e_i$, if $R_f < R_i$ then $e_o < e_i$.

The TR-20 allows 3 combinations of gains (without the use of pots). For any amplifier the user may select either 10^5 ohm resistors, 10^4 ohm resistors, or a combination of the two.

Examples of these cases are shown in figure 3. (Page 8)

c. Amplifier as a summer

The amplifier may be used as a summer by inputting more than one voltage. Under the usual arrangement the feedback resistor, R_f, will be 10^5 ohms. This will allow the voltages being summed a choice of two gains of (1) and two gains of (10). See figure 4-a.

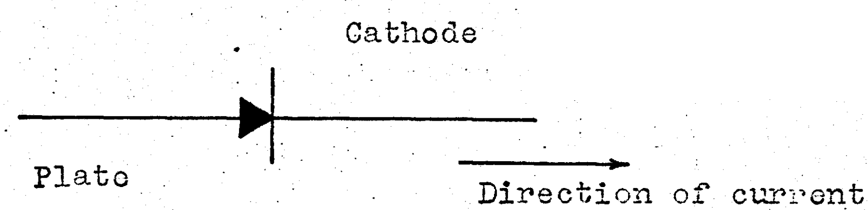
d. Amplifier as an integrator

When the feedback resistor is replaced by a capacitor, the amplifier circuit becomes an integrator. (Figure 4-b) The TR-20 has capacitor values of 10^{-5} farads <u>only</u> available for integration.

C. Nonlinear Components

1. Diodes

The diodes are generally solid state with a forward resistance on the order of about 100 ohms. Basically diodes are analogous to the operation of a check valve in pipe flow. As long as the plate voltage is greater than the cathode voltage, the diode conducts. When the plate voltage becomes smaller, the diode represents infinite resistance.



DIODE SYMBOL

2. Multipliers

The quarter square multiplier is basically a function generator which does the following operation.

$$xy = \frac{1}{4}\left[(x+y)^2 - (x-y)^2\right]$$

The method of generating $(x \pm y)^2$ for quarter square multiplication is similar to the principle used in all the fixed diode function generators (such as x^2, log x, sin x, etc.). The function generated may be referred to as $f_{(x)}$, and the input, for the QSM, are (x+y) and (x-y).

Basically the QSM generates a curve which approximates the square law curve. Practically, however, the QSM takes two variable inputs, x and y, and gives an output xy.

3. Sine-Cosine Generator

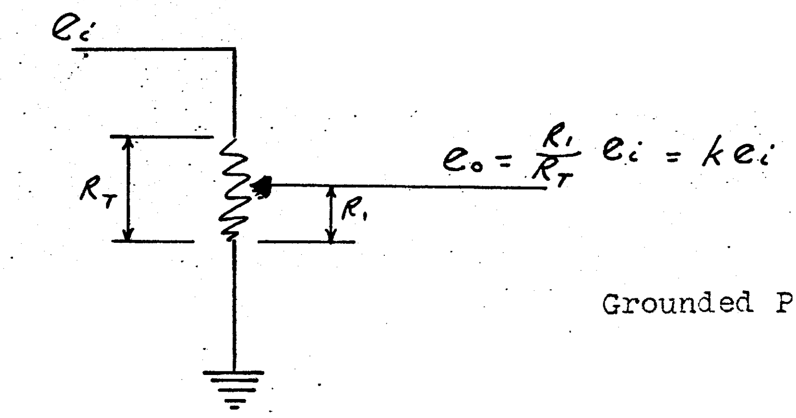
The Sine-Cosine Generator is similar to the QSM in that it is basically a function generator. It provides a fixed function of variables in all four quadrants ($\pm$ 180°) for sine-cosine.

This function may be easily generated directly on the computer, however, it requires the use of several integrators and amplifiers. Figure 6. (Page 11)

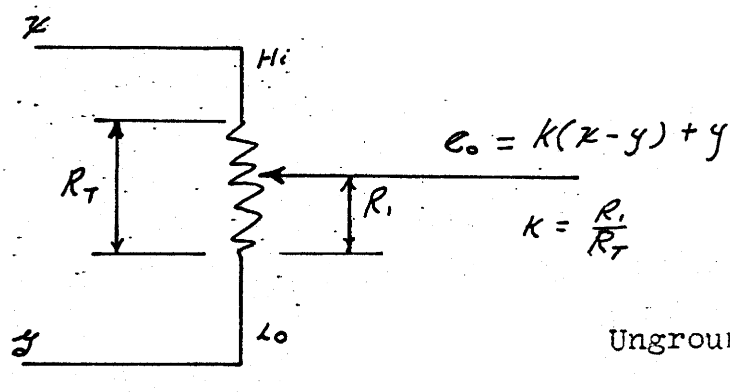
4. Comparator

The electronic comparator is a versatile high-speed switching device. When an input reaches a certain specified voltage a switch is activated and another input is substituted for the original input.

FIGURE 1

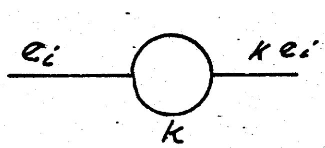


Grounded Potentiometer

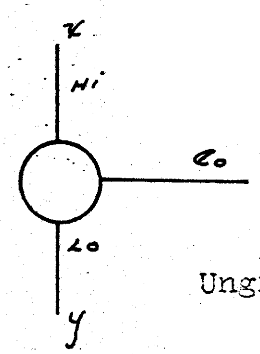


Ungrounded Potentiometer

PROGRAMMING SYMBOLS



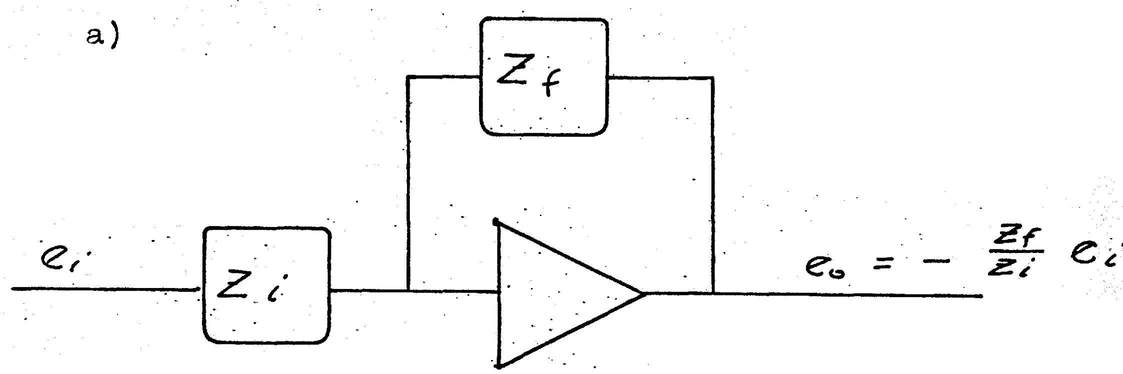
Grounded Potentiometer



Ungrounded Potentiometer

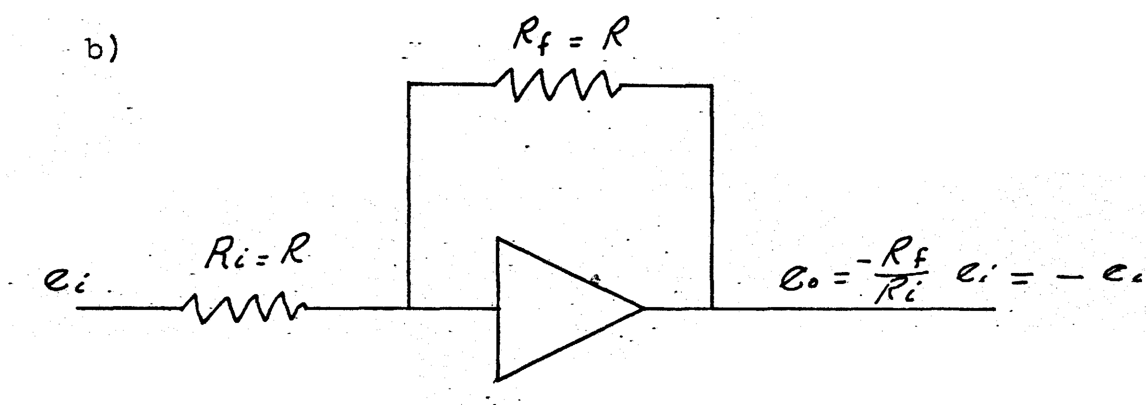
FIGURE 2

a)



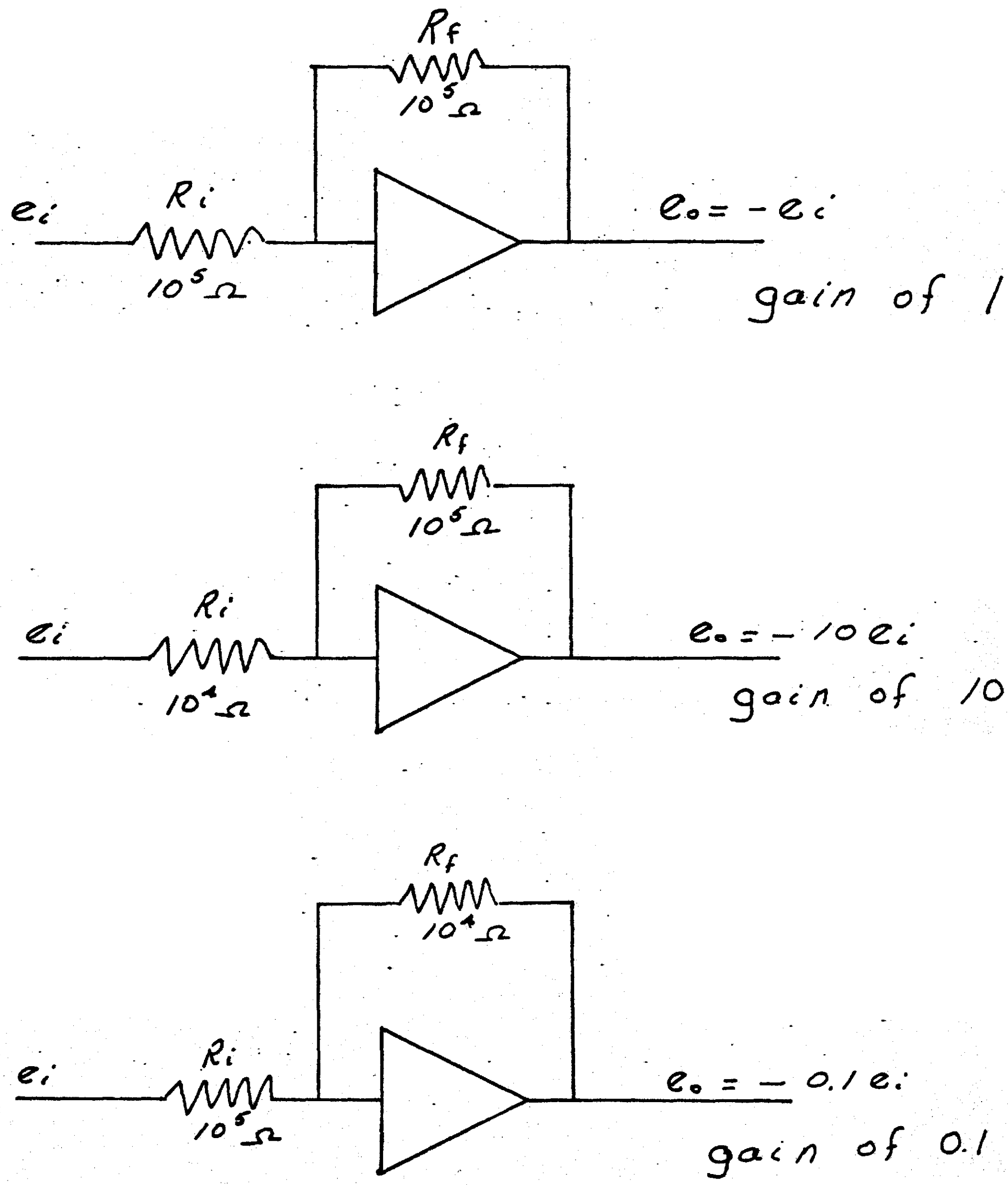
AMPLIFIER IMPEDANCE SCHEMATIC

b)



INVERTER

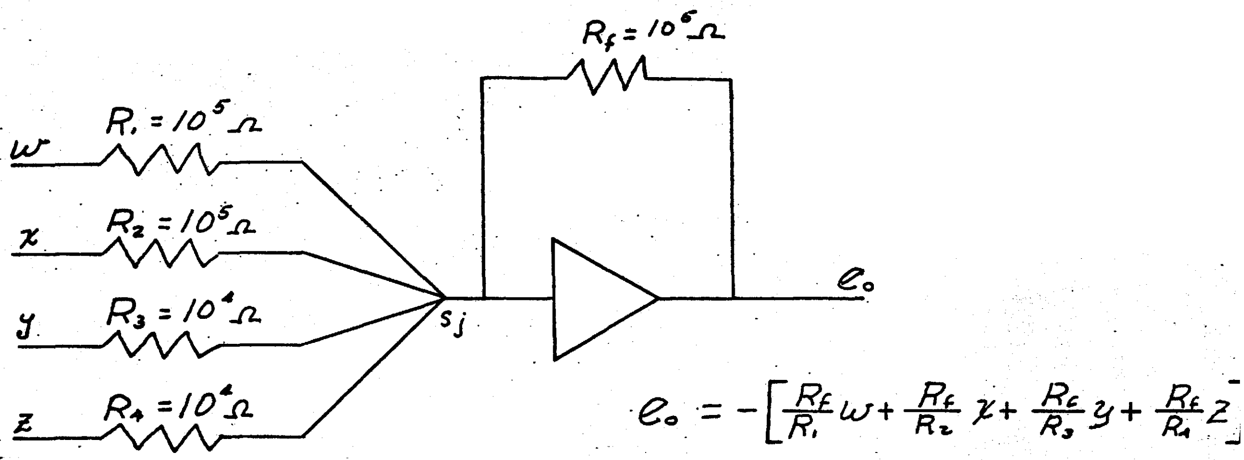
FIGURE 3



GAINS

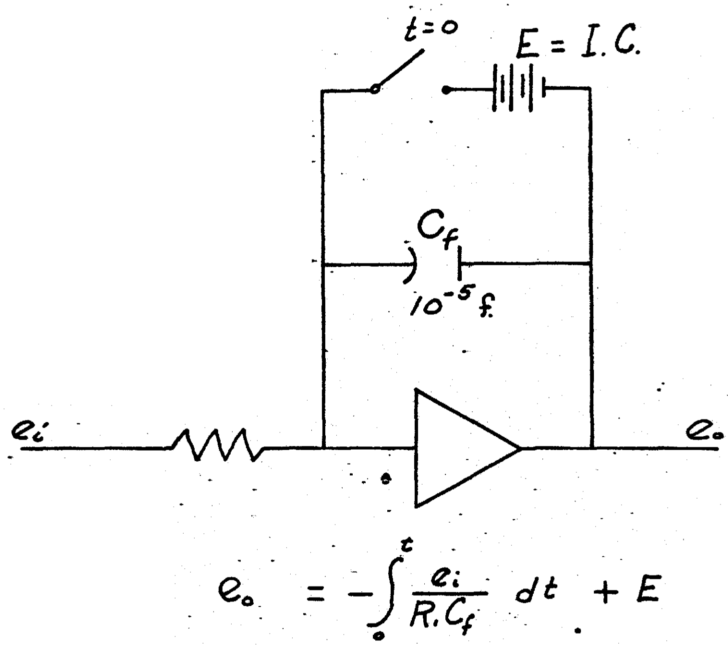
FIGURE 4

a)



SUMMING AMPLIFIER

b)



INTEGRATOR

FIGURE 5
PROGRAMMING SYMBOLS

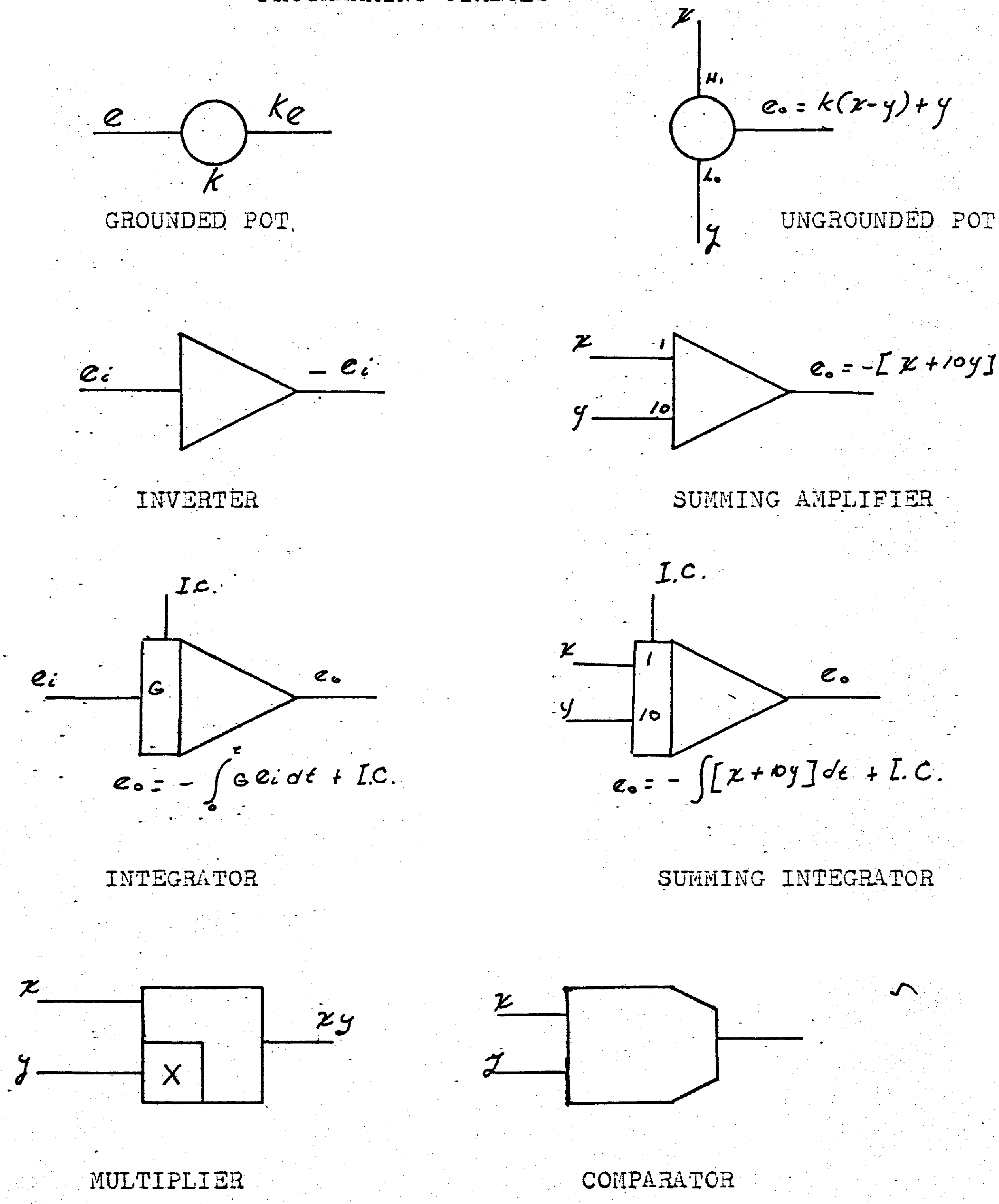
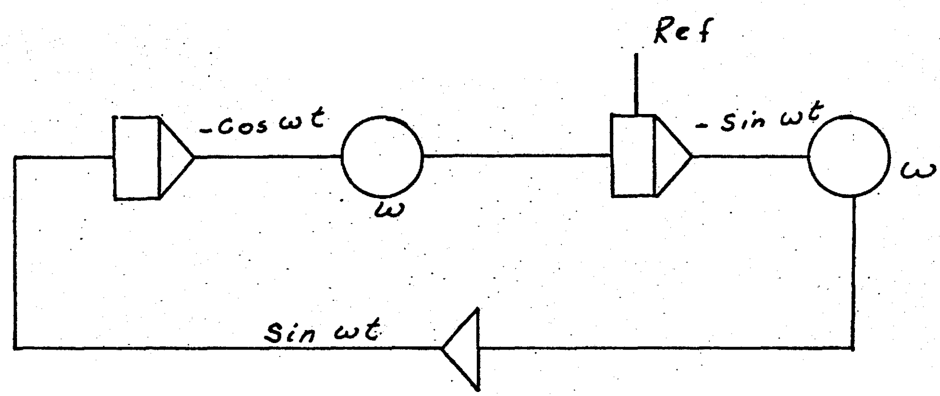


FIGURE 6

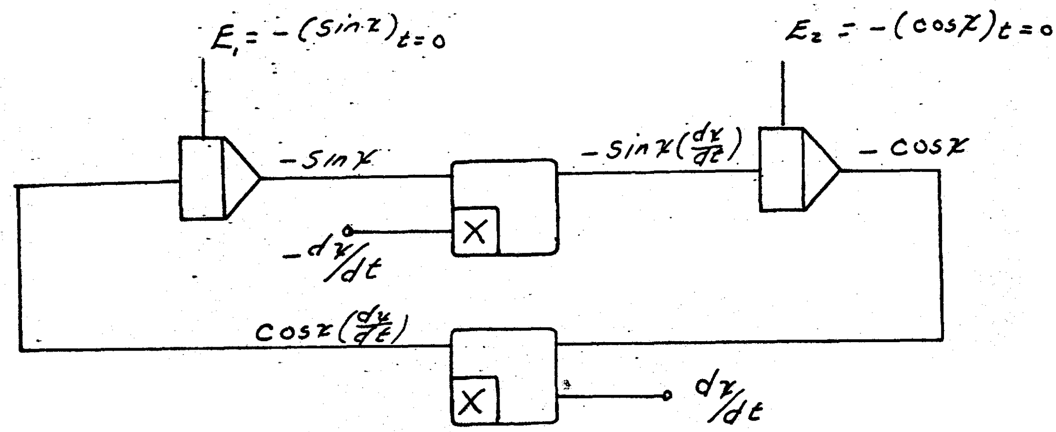
a)



Sine t - Cosine t

GENERATOR

b)



Sine x - Cosine x

GENERATOR

MANUAL CONTROLS AND READOUT FACILITIES

A. Control Panel

1. Overload Alarm - The lamps are illuminated whenever their associated amplifier is overloaded.

2. Power ON-OFF Switch - Controls application of primary a-c power to the power supply of the computer.

3. Mode Control Switch - Controls the operational mode of the computer. Positions are Reset, Hold, and Operate.

4. Voltmeter Function Switch - Control voltmeter operation. Positions are POT BUS, NULL, VM, AMPL, and BAL.

5. NULL POT and Reference Selector Switch - Used in conjunction with the voltmeter to measure voltages by the null comparison method.

6. VM Jack - Provides for external inputs to the voltmeter when the Voltmeter Function Switch is in the NULL or VM position.

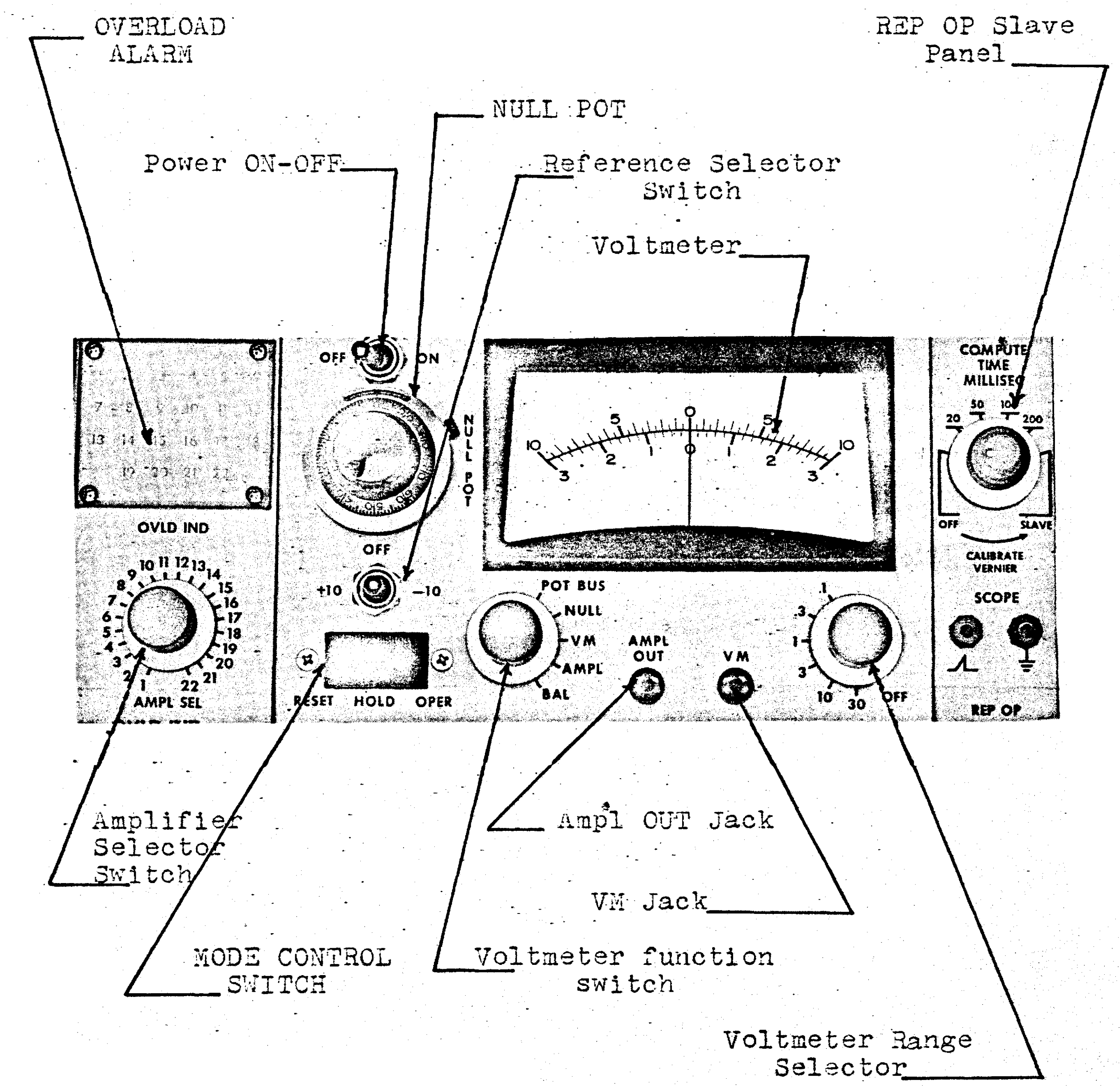
7. Amplifier Selector Switch - Selects an amplifier for output monitoring or balancing.

8. AMPL OUT Jack - Connected to the wiper of the AMPL SEL switch; facilitates connecting any amplifier output to external monitoring or measuring equipment.

9. REP OP SLAVE PANEL - Provides fixed time scaling from 200 to 20 milliseconds. Must be on for display on oscilloscope.

10. Voltmeter Range Selector - Selects range of voltmeter.

TR-20 CONTROL PANEL



B. Oscilloscope

1. Patch Panel Display Unit - Outputs from amplifiers are patched into y_1, y_2, y_3, and y_4. There should *always* be a jumper between x_d and /_ on this unit (see first pink page in "EAI-TR-20 Computer - Operators Reference H andbook").

2. Channel Switch on Scope

   a. Large knob - Selects output desired i.e. y_1, y_2, y_3, or y_4; or if in "all" position will give output from y_1, y_2, y_3 and y_4 simultaneously.

   b. Small knob - Controls horizontal location of display on scope.

3. Function Switch on Scope

   a. Large knob

      (1) Off - Removes power from scope.

      (2) Sweep - Connects ramp function from timing unit to horizontal input.

      (3) X Plot - Connects patch panel display terminal y_4 to horizontal input.

   b. Small knob - Controls brightness of image.

C. x-y Plotter

1. Patch Panel Display Unit - Output from amplifier is patched into y_1. Output from generated ramp function is patched into x_1. There should *still* be a jumper between x_d and /_ on this unit.

2. Plotter Control Panel

a. Mode Select Switch

(1) VAR - When in this mode the SCALE FACTOR control provides a means of obtaining any value of scale factor from the selected fixed value down to zero deflection.

(2) CAL - Permits the operator to calibrate the plotter deflection in reference to the computer machine unit.

(3) ZERO - The input signal is removed from the system. This allows the setting of X-Zero or Y-Zero.

(4) FIXED - The scale factor applied to the input signal is controlled solely by the RANGE switch.

b. SCALE FACTOR Control - Used in conjunction with either the VAR or CAL Mode Select Switch to obtain scale factors for output.

c. RANGE - Permits the selection of eight indicated scale factors.

("EAI - Computer Module, Model 12.834, 1110 Variplotter System")

PROGRAMMING METHODS

A. I ntroduction

The easiest programming method is the so-called boot strap method. There are several steps in programming this method which may be of interest.

Steps in programming and magnitude scaling:

1. Determine equations of math model.
2. Solve differential equations for highest order derivative. Write expressions for lower order derivatives.
3. Introduce appropriate scale factors into equations so that equations are in terms of machine units.
4. Sketch a circuit diagram.
5. Estimate the maximum anticipated values of the physical quantities and determine the scale factor magnitude.
6. Write the circuit equations from the circuit diagram. These equations will be in terms of voltages, resistors, capacitors, potentiometer settings and circuit constants.
7. Determine the required values for constant input voltage, etc.
8. Show the values determined in step 7 next to the appropriate symbols in the circuit diagram.

B. Programming

An example of this for programming only is the following equation:

$$x = ce^{-kt}$$

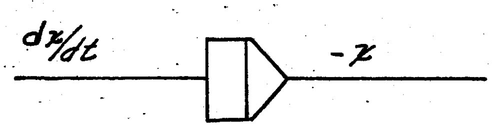
upon differentiation we have

$$dx = -kce^{-kt}dt$$

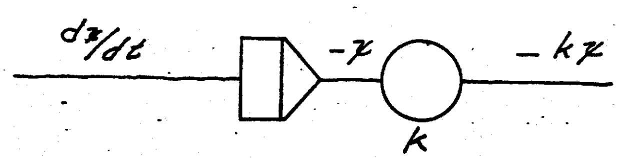
or $$dx = -kxdt$$

$$\frac{dx}{dt} = -kx$$

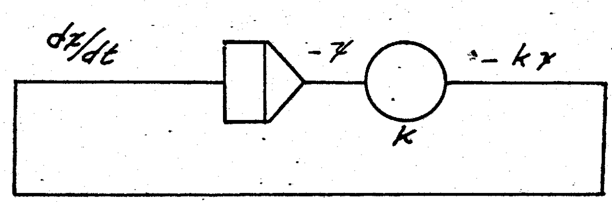
Obviously the component which best represents the relationship between the two variables is the integrator.



Now if we insert a pot



which is the solution to dx/dt. Now connecting the output and the input

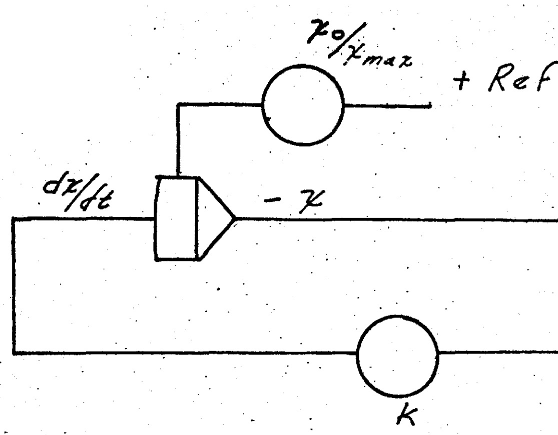


Returning to our original equation

$$x = ce^{-kt}$$

we see that when t = o, x = c.

Therefore, our initial condition is $x_o = c$



Another example of programming, again without magnitude scaling, is the solution of the Euler equation for column buckling (pin-ended).

$$\frac{d^2y}{dx^2} = \frac{M}{EI} = -\frac{P}{EI}y$$

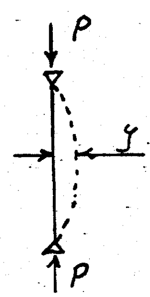
where M is the moment

E is the modulus of electricity

I is the moment of inertia

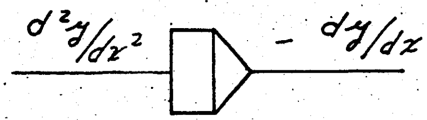
P is the compressive load on the column,

y is the deflection of the column.

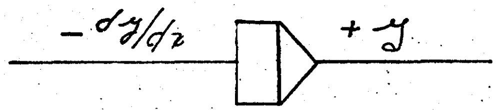


This may be programmed using the "boot strap" method as follows:

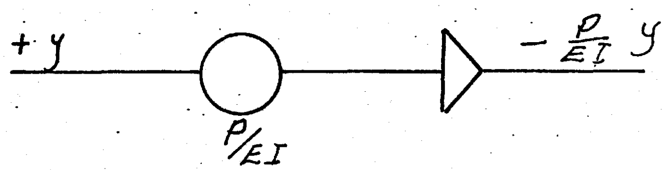
Integrate $\frac{d^2y}{dx^2}$ to get $-\frac{dy}{dx}$



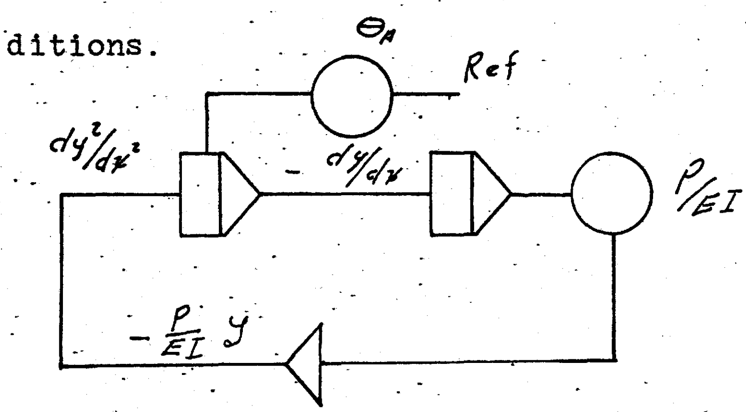
Integrate $\frac{-dy}{dx}$ to get +y



Put +y through a pot and an inverter



connect the various components and add initial conditions.

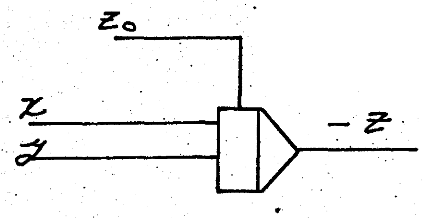


For the given problem the slope of the colum will be zero in the center of the column. With this in mind e_a can be located by trial and error and the problem will be solved.

C. Magnitude scaling

Assuming that the following equation has been programmed as shown, magnitude scaling would *begin* with the choice of maximum values.

$$\frac{dz}{dt} = x + y$$



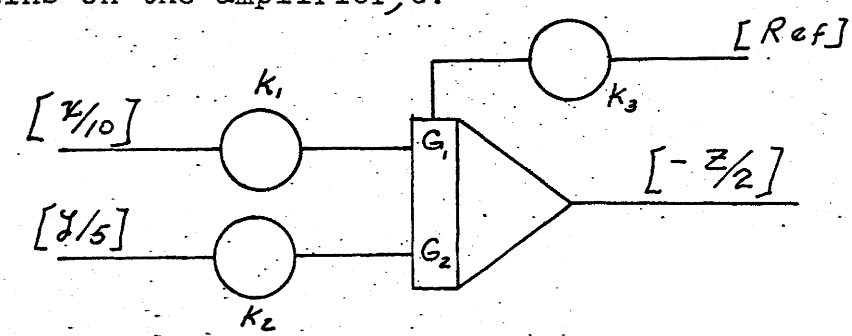
Now assume that the following maximum values are selected:

$x_{max} = 10$

$y_{max} = 5$

$z_{max} = 2$

We will say, therefore, that the computer variables are $[x/10]$, $[y/5]$, and $[-z/2]$. We will have also some "pot" settings, k, and gains on the amplifier, G.

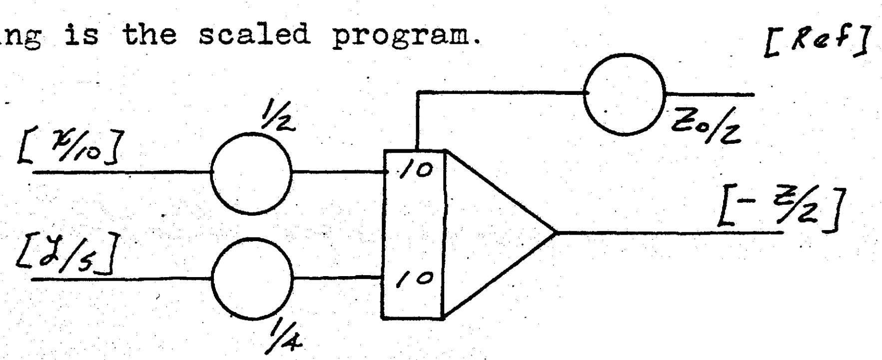


Values of k_1, G_1, k_2, G_2, and k_3 may be found by dividing the denominators of the input by the denominators of the output.

$k_1G_1 = 10/2 = 5$

$k_2G_2 = 5/2 = 2.5$

Remembering that k_1 and k_2 must be less than 1 and that the most logical choice of gain is either 1 or 10 it is obvious that the following is the scaled program.

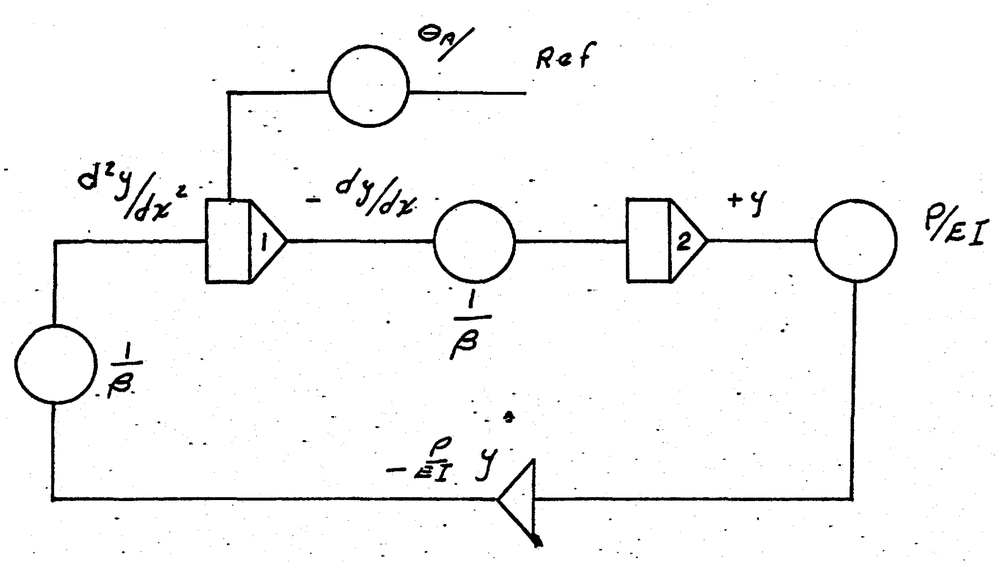


k_3 is the ratio of the initial condition to the maximum value for the output.

D. Time scaling

Occasionally it is necessary to increase or slow down the time necessary for the computer to solve various problems. This may be done for certain fixed values directly on the computer for the rep-op and plotter, or it may be accomplished by changing the input to the program integrators.

The direct method will be discussed in the section on adjusting equipment. Time scaling is usually accomplished by correcting the input to the integrators. In the previous example of the Euler Equation problem it would be scaled in the following manner.



Notice that the initial condition to integrator 1 was not time scaled. Only direct inputs to integrators are scaled by same factor β.

EXAMPLE PROBLEMS

The following problems are given as examples of the previously discussed techniques.

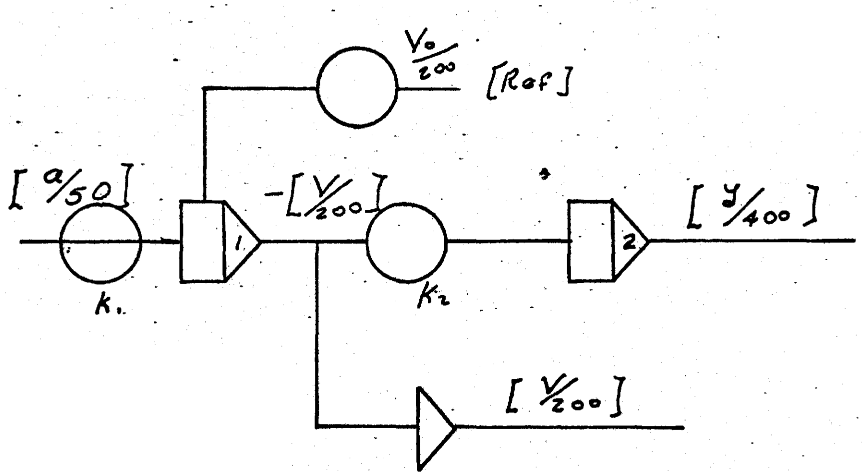
1. A body is projected vertically upward at an initial velocity of 128.8 ft/sec. Solve for the velocity and displacement at t secs.

Solution: If air resistance is neglected, the problem-variables are the acceleration, velocity, and displacement.

Now maximum values of the variables must be selected. The maximum value of the acceleration due to gravity is, of course, 32.2 ft/sec^2. However, in order not to "crowd" our solution we will let A_{max} be 50 ft/sec^2. For the same reason we can set V_{max} as 200 ft/sec. The maximum displacement may be solved for:

$$y_{max} = \frac{V^2_{max}}{2A_{max}} = \frac{4 \times 10^4}{10^2} = 400 \text{ ft.}$$

Preliminary circuit diagram:



Now Gains and Pot settings must be solved.

$$K_1 = \frac{50}{200} = .25$$

but a = 32.2 ft/sec^2, therefore

$$\frac{a}{50} = \frac{32.2}{50} = .644 \text{ (ref)}$$

$$k_1 = (.25)(.644) = .161$$

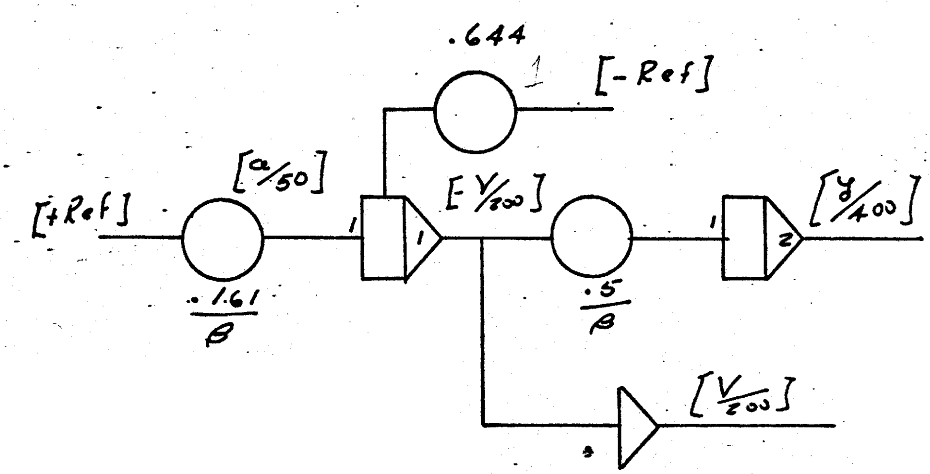
and k_2 is found as

$$k_2 = \frac{200}{400} = .5$$

Finally $V_0/200$

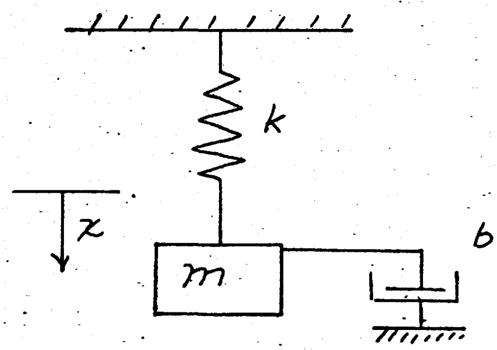
$$V_0/200 = \frac{128.8 \text{ ft/sec}}{200 \text{ ft/sec}} = .644$$

which gives the following diagram.



Example 2.

The spring-mass-damping apparatus shown below has the following initial values: $k = 100$, $m = 25$, $x_o = 4$, $\dot{x}_o = 0$ It is desirable to try several values of the damping co-efficient, b, in order to study the effects of damping on this system. Consequently, the following values will be used for b: 50, 25, 12.5, 6.25.



Solution: Note that the system has been displaced 4 units from its equilibrium position.

The equation for the above system is a second order equation.

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0$$

This equation is found by summing forces in the x direction i.e.

$$F_1 = ma = m \frac{d^2x}{dt^2}$$

$$F_2 = bv = b \frac{dx}{dt} \quad \text{(a function of velocity)}$$

$$F_3 = kx \quad \text{(Hookes law)}$$

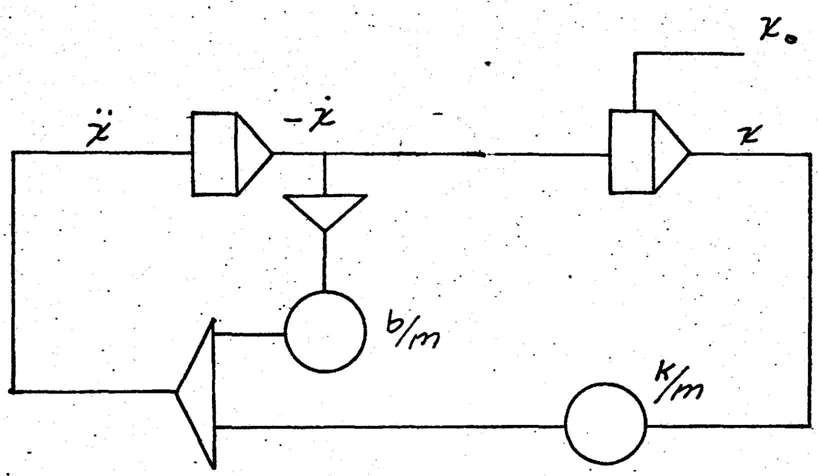
Now solving the equation for the highest derivative:

$$\frac{d^2x}{dt^2} = -\frac{b}{m}\frac{dx}{dt} - \frac{k}{m} x$$

or

$$\ddot{x} = -\frac{b}{m}\dot{x} - \frac{k}{m} x$$

The preliminary circuit diagram will now be shown:



The problem variables are acceleration, velocity, and displacement; it is necessary to find the maximum values for these.

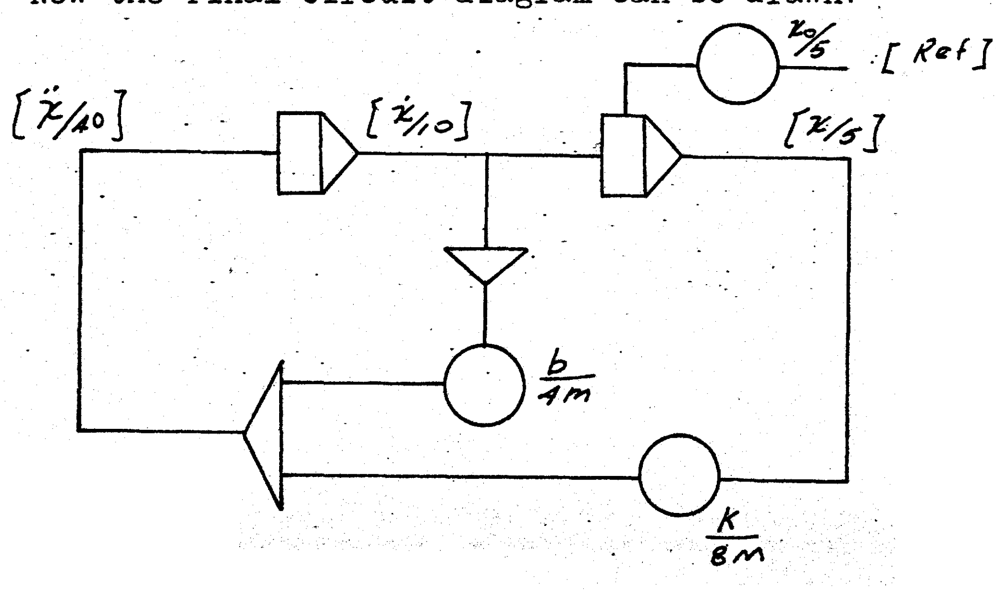
Let $o_{max} = 40$

$\dot{v}_{max} = 10$

$x_{max} = 5$

or $\frac{\ddot{x}}{40}$, $\frac{\dot{x}}{10}$, $\frac{x}{5}$

Now the final circuit diagram can be drawn.



Where $k/m8 = \frac{100}{(25)(8)} = .5$ 1.0

and $b/m4 = \frac{50}{100} = .5$

$= \frac{25}{100} = .25$ .34

$= \frac{12.5}{100} = .125$

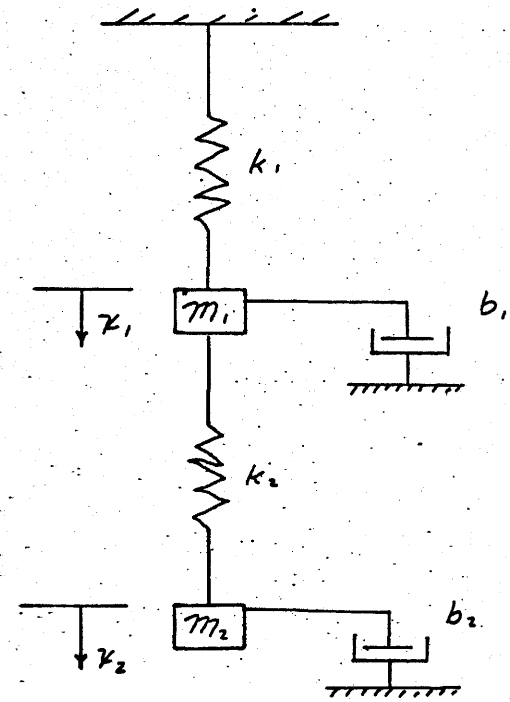
$= \frac{6.25}{100} = .0625$

and $x_o/5 = 4/5 = .8$ 1.0

Note that as b approaches o the system approaches an oscillator.

Example 3.

A two degrees of freedom problem will now be analyzed.



Analysis of the system gives the following equations:

(1) $m_1\ddot{x}_1 = k_2(x_2-x_1) - k_1(x_1) - b_1\dot{x}_1$

(2) $m_2\ddot{x}_2 = F(t) - k_2(x_2-x_1) - b_2\dot{x}_2$

or

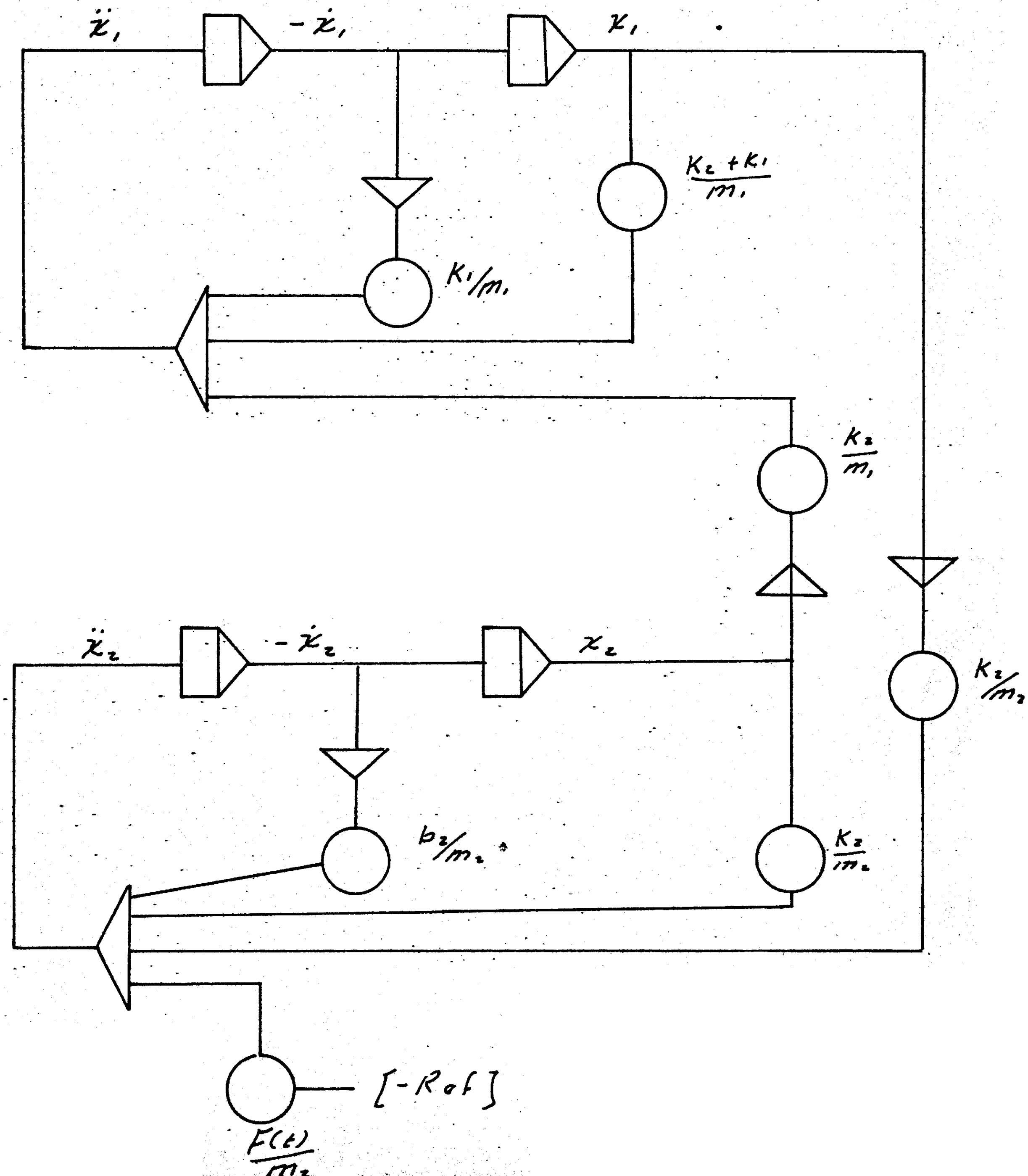
(1) $\ddot{x}_1 = \frac{k_2}{m_1}(x_2-x_1) - \frac{k_1}{m_1}(x_1) - \frac{b_1}{m_1}\dot{x}_1$

(2) $\ddot{x}_2 = \frac{F(t)}{m_2} - \frac{k_2}{m_2}(x_2 - x_1) - \frac{b_2}{m_2}\dot{x}_2$

The problem is programmed by first drawing the diagram for equation (1), then the diagram for equation (2), and then connecting shared members.

The problem is scaled by assuming that the two problems are separate. By scaling the bottom first an estimate of $k_2\ x_2$

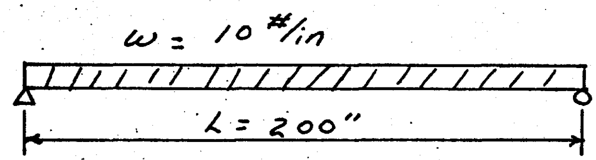
is found which may be used in the first equation as the driving force.



Example 4.

Beam deflection problem.

Solve the given beam for its shear, moment, slope and deflection.



Solution:

The problem variables are as follows:

$$d^4y/dx^4 = -w$$

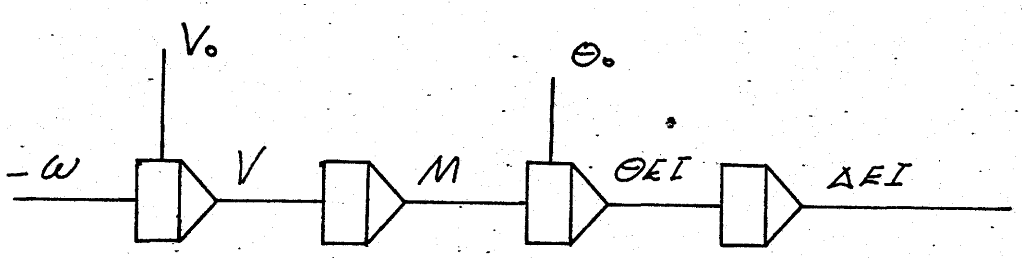
$$d^3y/dx^3 = V$$

$$d^2y/dx^2 = M$$

$$dy/dx = EI\theta$$

$$y = EI\Delta$$

The preliminary circuit diagram will be as follows:



The problem will have to be solved for maximu values, i.e.

Let

$$w_{max} = 10 \text{ lbs./in}$$

$$L_{max} = 300 \text{ in}$$

Therefore

$$V_{max} = 1000 \text{ lbs.}$$

$$M_{max} = 100{,}000$$

$$EIy_{max} = \frac{-wx^4}{24} + \frac{wlx^3}{12} - \frac{wl^3x}{24}$$

$$EIy_{max} \approx 2x10^8$$

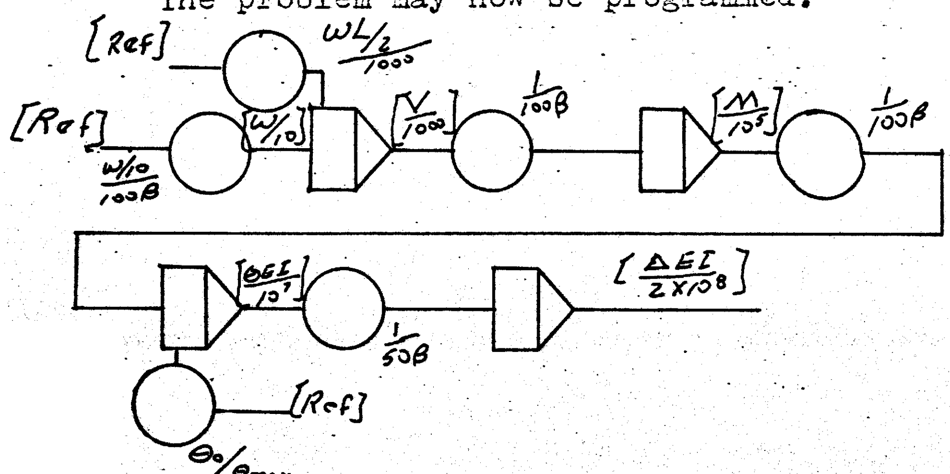
The value of EI_{max} may be found by trial and error if it is remembered that because of symmetry the slope angle is o in the center of the beam.

Computer variables

$[w/10]$, $[V/1000]$, $[M/10^5]$, $\left[\frac{\Theta EI}{\Theta max}\right]$, and

$\left[\frac{EIy}{2x10^8}\right]$

The problem may now be programmed.



EXPANSION OF COMPUTER AND LAB FACILITIES

I. Computer

As of this writing the computer now has the following components.

Amplifiers - 16

Potentiometers

a. Carbon - 10

b. Wire-wound - 6

Integrators - 6

Multipliers - 2

Sin - Cos generator - 1

Electronic Comparator - 1

REP-OP - 1

The following components are recommended for future expansion of the present TR-20.

Amplifiers - 4

Potentiometers -

a. Wire-wound - 8

Integrators - 2

Multipliers - 1

VDFG - 2

Relay Comparator - 1

It is suggested that the integrators and the VDFG be the next units purchased. T his should give a basic capability for problems of structual nature. It is suggested that as soon as economically possible after the purchase of the VDFG and integrator units that another basic TR-20 be purchased. An alternative to this suggestion would be the purchase of extra components; since the interchange of components on the TR-20 is a relatively simple affair. This should be the most economical method of fully utilizing our present TR-20.

II. Personnel

It is suggested that a graduate assistant be placed in charge of the computer laboratory. He should be desked in or near the lab and be responsible for the general condition of the lab and "watchdog" of the computer. He should be cognizant of the principles of analog computation and programming. Among his duties should be the teaching of "short courses", preparation of demonstrations for the faculty, and the upkeep of the machine itself.

III. Laboratory

The laboratory should contain the following items:

Benches or tables for the computer and accessories

Bench or table for prepatching

Storage area for patch boards, bottle plugs, patch cords, etc.

Blackboard

Desk for graduate student.