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"AN ANALYSIS OF MULTI-LEVEL ENCODING"

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AN ANALYSIS OF MULTI-LEVEL ENCODING

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Abstract *- It has been suggested that the capacity of a digital magnetic recorder can be increased beyond that normally obtainable, by using linear multi-level encoding, rather than the normal saturated two-level system. The technique is analyzed here by comparing its performance to that of normal two-level recording. The multi-level technique offers little, if any, improvement of the channel capacity in currently used systems, and is eventually limited by amplitude irregularities.*

MULTI-LEVEL ENCODING TECHNIQUE

The general idea behind the application of multi-level encoding is that several bits of data can be encoded by only one transition on the medium, if several discrete magnetization levels can be utilized. For example, suppose that, instead of only using the two magnetization levels of +M and -M on the medium, we use +M, +M/3, -M/3, and -M. Each level can then represent two data bits (00,01,10,11), and yet the flux-transition rate is the same as normally used. This is clearly a data packing density improvement of a factor of two. The technique can be extended so that eight levels can represent three data bits, giving a data packing density improvement of three, and so on. Note that the capacity of the machine is also improved by the same factor, if the transition rate can in fact be maintained at the original rate.

In practice it will not be possible to maintain the same transiton rate as the two-level system, because as more levels are used, the playback amplitude reduces, and there may be insufficient signal-to-noise ratio (SNR) to produce an acceptable error-rate. Reducing the transition rate would help to compensate for this problem in two ways:

(1) A lower transition density means less intersymbol interference upon playback, and therefore higher readback amplitudes.

(2) A lower transition rate means lower overall frequencies during playback, allowing for a lower low-pass filter bandwidth, which results in less Gaussian and high frequency noise.

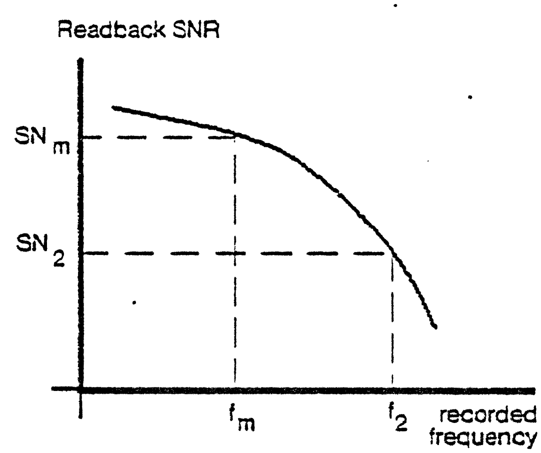

Fig. 1 SNR response of digital recorder

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In general therefore, it is proposed that multi-level encoding can be used to enhance the performance of a digital recorder as follows:

(a) Take an existing or proposed digital recorder and determine its operating point on a graph of "readback signal-to-noise ratio" versus "transition rate" (points f_2 and SN_2 in Fig. 1).

(b) Change to a reduced transition rate (f_m), yielding an improvement in the signal-to-noise ratio (now SN_m).

(c) Nullify the improvement in SNR by utilizing more recorded levels, so that the resulting SNR on readback is SN_2 again, yielding the same error-rate as the original machine (it has been shown [1] that maintaining a fixed SNR will yield essentially the same ultimate performance, or error-rate, when comparing different coding techniques in a given system).

As an example, consider the simplified case shown if Fig. 2, where a machine is originally operating with two-levels at f. Now, channel capacity (C) is given in terms of transition rate (f) and the number of levels (n) by:

$$C = f \times \log_2 n.$$

If the slope of the graph is 16dB/octave, and we change to a transition rate of f/2, then the resulting SNR is 7 x SN, which can be reduced back to SN by employing 8 levels:

$$\text{Old capacity} = f \times \log_2 2 = f$$

$$\text{New capacity} = (f/2) \times \log_2 8 = (f/2) \times 3 = 1.5 \times f$$

The capacity improvement is thus 50%.

ANALYSIS

Errors due to noise

Noise in a digital recording system has two main sources: (a)Electronics and Media noise, which both may be of Gaussian distribution, and are called additive noise, and (b)Amplitude variations in the readback signal, called multiplicative noise. The latter's distribution may be a Rayleigh or Rice function.

Both noise voltages will occasionally be of sufficient amplitude to be detected as a signal, which consequently is an error. The occurrence of these errors depends on the signal-to-noise ratio of the recording/readback channel, and upon the probability function of the noise sources.

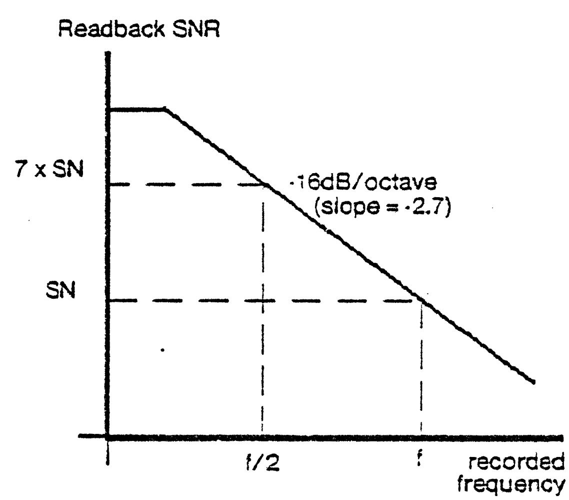

Fig. 2 SNR versus frequency (example)

Additive noise. - The maximum number of signal levels (n) is determined by the SNR according to (from Fig. 3):

$$n = (V_{signal} + V_{noise}) / V_{noise}$$
$$= SNR + 1 \qquad (1)$$

Alternatively, we can write:

$$SNR = [20 \times \log(n - 1)] + SNR_0 \qquad (2)$$

where the SNR's are now in dB, and SNR_0 is the signal-to-noise ratio required for a specified error-rate (BER). This is approximately 21 dB for a BER less than 10^{-6}, and we can calculate a simple table:

Number of levels	Required SNR
2	21.0 dB
4	30.5 dB
8	37.9 dB
16	44.6 dB

A change from a 2-level system to a 4-level system will require an increase of 9.5 dB in SNR, which may be achieved, for example, by reducing the bandwidth to one-half of the original value. There would be no improvement in the channel capacity in this case, indicating that the signal-to-noise ratio should have a minimum slope of -9.5 dB/octave in order to improve the capacity.

In general, let the initial channel capacity be $C = f \times \log_2 n$, and let us now decrease the transition rate from f to f/k.

If s = slope of the SNR versus transition rate curve (assumed linear), this bandwidth reduction will increase the SNR by k^s (note that $s = (\text{dB/octave} \times \log_2 10)/20$).

This increased SNR can be exactly offset by changing the number of signal levels from n to $((n-1)k^s + 1)$. Note that SNR is proportional to $(n-1)$, not n.

The new capacity is $(f/k) \times \log_2[(n-1)k^s + 1]$, and hence:

$$\frac{\text{New capacity}}{\text{Old capacity}} = \frac{1}{k} \cdot \frac{\log_2[(n-1)k^s + 1]}{\log_2 n}$$

For n (= original number of levels) = 2, and normalizing the capacity to the original value, this reduces to :

$$\text{Normalized capacity} = (1/k) \times \log_2 (k^s + 1) \qquad (3)$$

This equation yields the graphs of Fig. 4, which shows normalized capacity obtainable for different numbers of recorded levels, and for various slopes of the SNR curve.

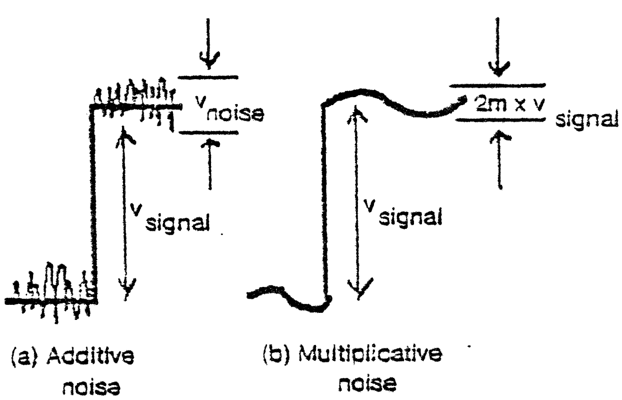

Fig. 3 Additive Noise and Multiplicative Noise

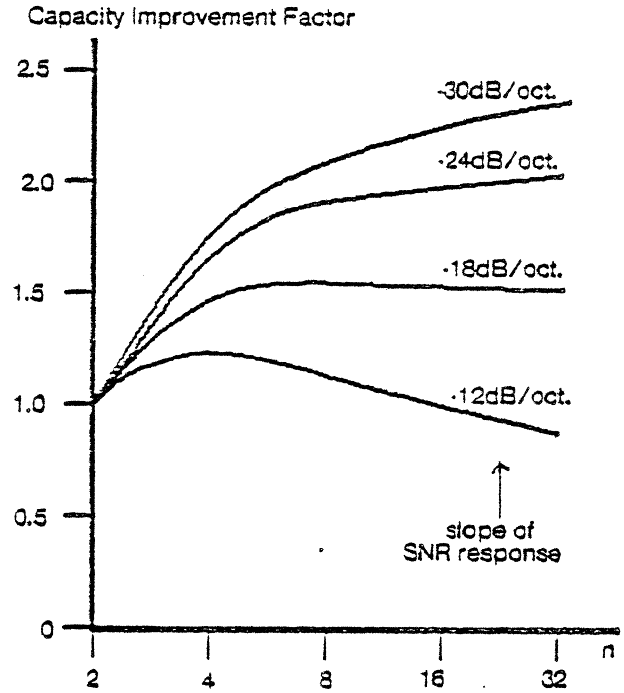

Fig. 4 Capacity improvement versus Number of Levels(n), for various slopes of the SNR response.

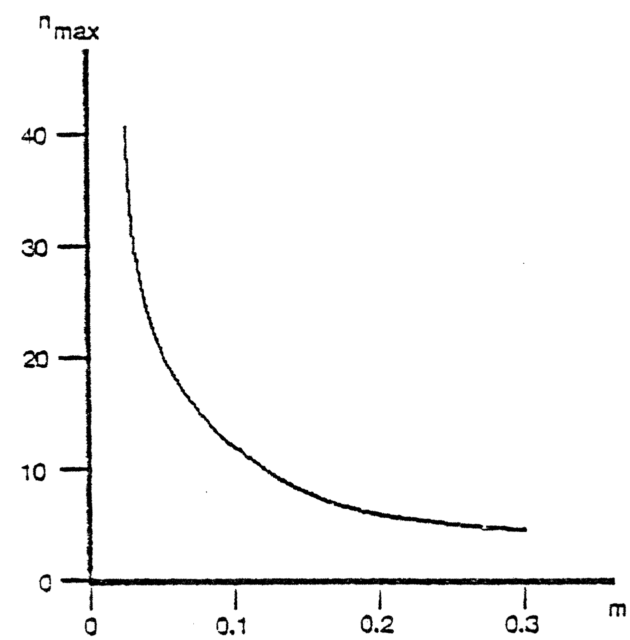

Fig. 5 Limitation of maximum number of levels (n) by amplitude modulation(m)

Multiplicative Noise - Amplitude variations will occur in all tape or disk drives due to non-uniform coating thickness and varying degrees of surface finsih. Variations will also occur in the spacing losses between the write/read transducers and the coating.

The effect of modulation can easily be determined for a simple multi-level system with equal increments between levels. From Fig. 3 we have, analogous to (1) :

$$n = 1 + 1/m \qquad (4)$$

where m is the amplitude modulation (peak value). This equation is plotted in Fig. 5.

In practice the modulation is, at best, 0.05 (=0.5 dB). This allows for only 21 levels, which in a practical system would probably be reduced to 16 (the nearest power of 2). This may be improved upon by using variable increments between recorded levels, but it is still unlikely that more than 32 levels could be used [2],[3].

However many levels are actually used, the amplitude modulation will detract from the SNR, such that normalized SNR = 1 - m(n-1), as plotted in Fig. 6. It is apparent that, with many recorded levels, it becomes possible for the SNR to be reduced considerably.

Not withstanding this reduction in SNR, it can be seen from Fig. 4 that a minimum of -18 dB/octave slope of the SNR curve is required in order to obtain a 50% improvement of the channel capacity; a 100% improvement requires -24 dB/octave, at and before the normal operating point for the machine that is to be modified.

EXPERIMENTAL SNR-CURVES

Experimental curves for several head/media combinations are shown in Fig. 7, plotting readback voltage versus recorded frequency. The normal operating point for a machine is typically where the read voltage has fallen by approximately 40%. The slope of all curves at this point is only approximately -10 dB/octave (and even less before this point, so that the effective value is still lower). With an allowance for a (rising) 3 dB/octave slope for noise voltage as a function of frequency, this means that the maximum slope of the SNR-versus-frequency curve is -13 dB/octave in practice. This is significantly less than the -18 dB/octave necessary to make multi-level encoding worthwhile.

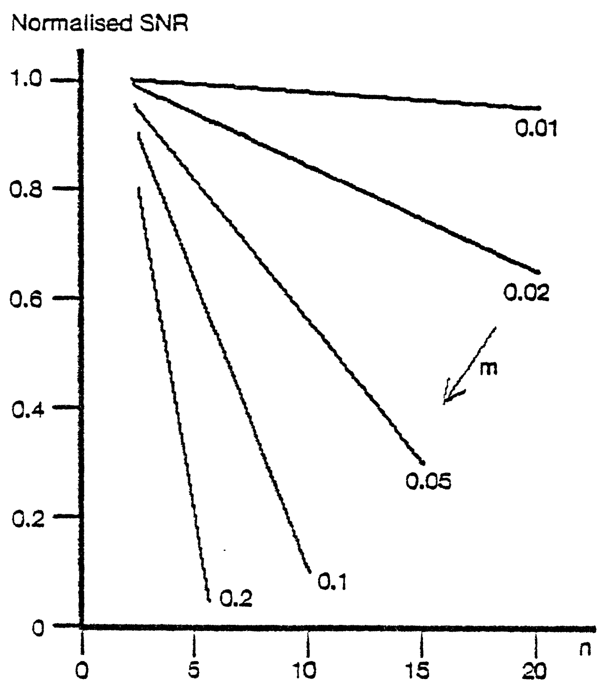

Fig. 6 Effect of modulation(m) and number of levels(n) on normalised SNR

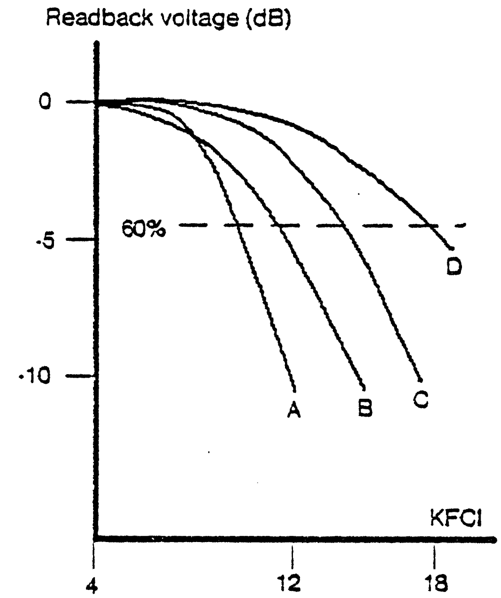

Fig. 7 Readback Amplitude versus Packing Density

A: 3370 thin-film head on oxide
B: Advanced ferrite head on floppy oxide
C: 3370 thin-film head on plated media
D: Advanced ferrite head on plated media

CONCLUSION

If the normal operating point of a given machine is such that the slope of the SNR-versus-frequency curve is greater than -18 dB/octave, then changing to multi-level encoding could yield a worthwhile improvement in capacity. However, in practice, this is not the case, the slope being typically -10 to -15 dB/octave, and so multi-level encoding is of no benefit, and would in all probability actually degrade the performance of the machine.

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